The Huntington-Hill Method – Version 1

Lecture 23
Section 4.5

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Outline

The Huntington-Hill Method

- In 1929, Congress set the size of the House of Representatives at 435 members.
- In 1941, Congress adopted the Huntington-Hill method for apportioning the seats in the House.
- Both laws remain in effect and will remain in effect for the foreseeable future.

The Huntington-Hill Method

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- There are two ways to apply the Huntington-Hill method.
- The first method, described in the textbook, involves guessing a modified divisor in a way similar to Jefferson's, Adams's, and Webster's methods.
- Therefore, it is not quite an algorithm.
- The second method, which is the one used by the government, involves no guesswork, but it may take (much) longer to compute.
- It is an algorithm.

Outline

The Huntington-Hill Method – Version 1

- Compute the standard quotas q_i for each state, as in the other methods.
- Round off the standard quota for each state by the following method.
  - Let L be the lower quota and U be the upper quota.
  - Compute the cutoff as $\sqrt{LU}$.
  - If $q_i < \sqrt{LU}$, then round down. Otherwise, round up.
  - The rounded value is the number of seats for that state.
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  - The rounded value is the number of seats for that state.
- If the total number of seats is not M, then choose a modified divisor and repeat the procedure.
- This method guarantees that each state gets at least one seat. How so?

Huntington-Hill Cutoffs

Comparison of Cutoffs

Lower Quota	Upper Quota	Huntington-Hill Cutoff	Traditional Cutoff
0	1	$\sqrt{0 \cdot 1} = \sqrt{0} = 0.000$	0.5
1	2	$\sqrt{1 \cdot 2} = \sqrt{2} = 1.414$	1.5
2	3	$\sqrt{2 \cdot 3} = \sqrt{6} = 2.449$	2.5
3	4	$\sqrt{3 \cdot 4} = \sqrt{12} = 3.464$	3.5
4	5	$\sqrt{4 \cdot 5} = \sqrt{20} = 4.472$	4.5

Example

Example (Example – Version 1)

- The populations of three states are $3, 7$ and 10 million people, respectively.
- The total number of seats apportioned to those states is 7.
- Use Version 1 to determine how many seats each state should get.

Example

Example (Example – Version 1)

- The total population is $P = 20$.
- The number of seats is $M = 7$.
- The standard divisor is $\text{SD} = \frac{20}{7} = 2.857$.

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3					
B	7					
C	10					

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	1.05				
B	7	2.45				
C	10	3.50				

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	1.05	1	2		
B	7	2.45	2	3		
C	10	3.50	3	4		

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	1.05	1	2	$\sqrt{1 \cdot 2} = 1.414$	
B	7	2.45	2	3	$\sqrt{2 \cdot 3} = 2.449$	
C	10	3.50	3	4	$\sqrt{3 \cdot 4} = 3.464$	

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	1.05	1	2	$\sqrt{1 \cdot 2} = 1.414$	1
B	7	2.45	2	3	$\sqrt{2 \cdot 3} = 2.449$	3
C	10	3.50	3	4	$\sqrt{3 \cdot 4} = 3.464$	4

Example

Example (Example – Version 1)

- The total number of seats apportioned is 8, so the "surplus" is -1.

Example

Example (Example – Version 1)

- The total number of seats apportioned is 8, so the "surplus" is -1.
- We need a larger divisor.

Example

Example (Example – Version 1)

- The total number of seats apportioned is 8, so the "surplus" is -1.
- We need a larger divisor.
- Let's try MD $= 3.2$.

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3					
B	7					
C	10					

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	0.937				
B	7	2.187				
C	10	3.125				

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	0.937	0	1		
B	7	2.187	2	3		
C	10	3.125	3	4		

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	0.937	0	1	$\sqrt{0 \cdot 1} = 0.000$	
B	7	2.187	2	3	$\sqrt{2 \cdot 3} = 2.449$	
C	10	3.125	3	4	$\sqrt{3 \cdot 4} = 3.464$	

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Example

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- The total number of seats apportioned is 6, so the “surplus” is $+1$.

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- The total number of seats apportioned is 6, so the "surplus" is $+1$.
- We need a smaller divisor.
- Let's try MD $= 2.86$.

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3					
B	7					
C	10					

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	1.049				
B	7	2.447				
C	10	3.498				

Example

Example (Example – Version 1)

State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
A	3	1.049	1	2		
B	7	2.447	2	3		
C	10	3.498	3	4		

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State	Pop	Standard Quota	L	U	$\sqrt{LU}$	Seats
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C	10	3.498	3	4	$\sqrt{3 \cdot 4} = 3.464$	4

Huntington-Hill–CA, TX, NY, VA, WV, and WY

Example (Huntington-Hill–CA, TX, NY, VA, WV, and WY)

State	Population	Std Quota	L	U	$\sqrt{LU}$	Seats
CA	39,776,830					
TX	28,704,330					
NY	19,862,512					
VA	8,525,660					
WV	1,803,077					
WY	573,720					

- The states CA, TX, NY, VA, WV, and WY currently have 131 congressional seats (CA 53, TX 36, NY 27, VA 11, WV 3, WY 1).
- Use the website to apply the Huntington-Hill method, version 1, to these states, using the 2018 estimated populations.

Outline

Assignment

Assignment

- Chapter 4 Exercises 43, 44, 45, 46, 49. Use Version 1.